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Squeezing of Many-Atom Radiation in an Optical Cavity

by

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Squeezing of many-atom radiation in an optical cavity

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Abstract

A method developed in terms of the SU(2) Lie algebra for the treatment of many-level atoms is now modified to treat the system of many two-level atoms interacting with a cavity field. Special attention is given to field squeezing up to fourth order. The effects of the number of atoms and detuning on the squeezing are investigated numerically for various initial field intensities. It is found that the atomic cooperation effect on the squeezing is closely related to the initial field excitation. When the initial excitation is strong, the cooperation enhances the squeezing, while in the case of weak excitation, it weakens the squeezing. The possibility of constructing many-atom micromasers is suggested.

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I. Introduction

In the squeezed state, a quadrature of the electric field can have smaller quantum fluctuation than in the coherent state.[1] The potential application of such squeezed light in low-noise communications and high-precision measurements has prompted a great deal of research in recent years.

By squeezing one usually means that the mean square deviation of a certain quadrature of the field is smaller than its corresponding value when the field is in the coherent state. It has recently been generalized by Hong and Mandel,[2] who introduced the concept of higher-order squeezing and demonstrated its existence in such processes as degenerate parametric down conversion, resonance fluorescence and harmonic generation.[3] In some cases, the higher-order squeezing may even be stronger than the ordinary second-order squeezing. In fact, the existence of fourth-order squeezing phenomena has been confirmed in theoretical studies of the single-photon and multiphoton Jaynes-Cummings (JC) model,[4] and in the anharmonic field.[5]

Even more interesting is the fact that in the multiphoton JC model there exists intrinsic higher-order squeezing.[3] Thus, higher-order squeezing has physical meaning and is not just a mathematical concept. As experimental techniques for measuring higher-order correlations in quantum optics are developing, it is significant to discuss high-order moments of the radiation field and the possibility of reduced fluctuation. It is expected from such discussions that higher-order correlation measurements may lead to the possibility of extracting the information more effectively from an optical signal.

The realization of the one-atom maser[6] makes it possible to observe predictions of the JC-model. One of the key factors that helps keep the micromaser in continuous operation is its ability to achieve the highest

possible Rabi frequency of the atom in the optical cavity. This is because faster Rabi oscillations allow more rapid energy exchange of photons between the atom and the cavity field. The Rabi frequency can be increased by increasing the number of interacting atoms with the cavity field.[7] In a micromaser, it is a relatively simple matter to adjust the current intensity of the atomic beam. When the beam intensity is sufficiently strong, there will be a bunch of atoms interacting simultaneously with the cavity field. It is therefore interesting to study an N-atom system interacting with a cavity field.

As the output power of a micromaser increases with increasing number of atoms, it appears that the many-atom micromaser is more attractive than the one-atom maser for practical purposes. If the mean interparticle distance is much smaller than the wave length of the cavity field, the system may be well-described by the Dicke model.[8] A study of the many-atom micromaser on the basis of this model is being carried out and will be published in the near future. It is hoped that such studies will induce the interest of experimentalists to develop many-atom micromasers as a powerful tool for the investigation of nonclassical phenomena of the cavity field.

Radiation processes from a system of many two-level atoms in a damped cavity have been examined recently for different initial conditions.[9] Collective effects on such properties as the atomic inversion, mean photon number and spontaneous emission caused by the presence of many atoms are explored. The purpose of this paper is to investigate how the atomic number and the strength of the coherent initial field influence fourth-order as well as second-order squeezing of the field, although second-order squeezing in the Dicke model has been briefly discussed.[10]

In Sec. II, we develop the theory in terms of the SU(2) Lie algebra. Numerical results are presented and discussed in Secs. III and IV. Finally, a brief discussion is given in Section V.

II. Theory

We consider a system consisting of a single-mode cavity field interacting with N atoms, each of which has two nondegenerate levels $|+\rangle$ and $|-\rangle$ with corresponding energies $\hbar\omega_+$ and $\hbar\omega_-$. Every atom is assumed to interact with the field of frequency Ω through a dipole transition only. For simplicity, we also assume that the mean interatomic distance is much smaller than the wavelength of the cavity field, so that all the atoms see exactly the same field. On the other hand, the atoms are still well separated such that their wave functions do not overlap and they see each other only through the coupling with the cavity field. Furthermore, we consider only single valence electron transitions in every atom. This means that the atoms can be treated as fermions.

The total Hamiltonian of our system can be written as

$$H = \sum_{\alpha=+,-} \hbar\omega_\alpha A_\alpha^\dagger A_\alpha + \hbar\Omega a^\dagger a + \hbar\lambda(A_+^\dagger A_- a + A_-^\dagger A_+ a^\dagger) \quad , \tag{1}$$

where we have defined

$$A_\alpha^\dagger A_\alpha = \sum_{i=1}^{N} A_\alpha^\dagger(i) A_\alpha(i) \tag{2a}$$

$$A_+^\dagger A_- = \sum_{i=1}^{N} A_+^\dagger(i) A_-(i) \tag{2b}$$

$$A_-^\dagger A_+ = \sum_{i=1}^{N} A_-^\dagger(i) A_+(i) \quad . \tag{2c}$$

The operator $A_\alpha^\dagger$ (A_α) creates (annihilates) an atom in the α-level, and i labels the atom. These operators obey the anti-commutation relation

$$[A_\alpha(i), A_\beta^\dagger(j)]_+ = \delta_{ij}\delta_{\alpha\beta} \quad . \tag{3}$$

We now define, for convenience, the operators

$$N_\alpha = A_\alpha^\dagger A_\alpha \tag{4a}$$

$$D^\dagger = A_+^\dagger A_- \tag{4b}$$

$$D = A_-^\dagger A_+ \quad . \tag{4c}$$

It is a simple matter to verify that the operators defined in (4) satisfy the following commutators:

$$[N_-, D^\dagger] = -D^\dagger \tag{5a}$$

$$[N_+, D^\dagger] = D^\dagger \tag{5b}$$

$$[D, D^\dagger] = N_- - N_+ \quad . \tag{5c}$$

Equations (5) characterize the U(2) Lie algebra. The first-rank Casimir operator of this algebra is just the total number operator of atoms

$$N = N_+ + N_- \quad . \tag{6}$$

It is easily seen that $[N,H] = 0$, i.e., the total number of atoms is a conserved quantity. Thus the operators $N_+ - N_-$, $D^\dagger$ and D defined in (4) constitute the SU(2) algebra.

Since N is conserved, it may be regarded as a constant in H. Therefore, the Hamiltonian H is practically composed of the SU(2) Lie algebra operators and photon operators. Consequently, if the wave function of the N-atom system belongs to a certain irreducible representation initially, it must remain in that same irreducible representation.

In the present paper, we limit our discussion to the case in which all the N atoms are initially in upper states. The atomic wave function corresponding to this case belongs to the full symmetry representation of SU(2), and the state of the atomic system remains in this irreducible representation all the time. The Hilbert space of SU(2) full symmetry representation can be defined by the basis vectors $|n_+,n_-\rangle$ which represent states with n_+ atoms in the upper level and n_- atoms in the lower level. Thus

$$N_+|n_+,n_-\rangle = n_+|n_+,n_-\rangle \tag{7a}$$

$$N_-|n_+,n_-\rangle = n_-|n_+,n_-\rangle \tag{7b}$$

$$N = n_+ + n_- \quad . \tag{7c}$$

In the space of this SU(2) full symmetry representation, the normalized state vector with an arbitrary number of atoms in the upper level can be derived, as is shown in the Appendix, from the initial state $|N,0\rangle$. The result is

$$|N-m,m\rangle = \sqrt{\frac{(N-m)!}{N!m!}}\, D^m\, |N,0\rangle \quad , \quad 0 \leq m \leq N \quad . \tag{8}$$

It can easily be verified that the system does not change its energy with interchange of any pair of atoms in the state.

We now introduce the total excitation operator

$$\Pi = N_+ + a^\dagger a \quad , \tag{9}$$

which commutes with the Hamiltonian and hence represents a conserved quantity. If initially the atoms are in the state $|N,0\rangle$ and the field is in $|n\rangle$, then at any time $t > 0$ the atom-field system can only be found in a state in that part of the Hilbert space spanned by the basis vectors

$$|N-m,m\rangle|n+m\rangle \quad . \tag{10}$$

Any arbitrary state in this subspace is characterized by the same eigenvalue N+n of the operator Π. Hence the Hilbert space can be divided into subspaces according to the total excitation number. All the state vectors in each subspace correspond to the same eigenvalue of Π. Consequently, the Hamiltonian operator can never connect states from different subspaces, and we can diagonalize the Hamiltonian in each subspace.

In the subspace defined by the total excitation number N+n, an arbitrary state $|\phi_n\rangle$ can be expressed as

$$|\phi_n\rangle = \sum_{m=0}^{N} C_{nm} |N-m, m\rangle |n+m\rangle \quad . \tag{11}$$

where the expansion coefficients C_{nm} satisfy the stationary-state Schrodinger equation

$$\sum_{m'=0}^{N} (H_{mm'} \delta_{mm'} + H_{mm'} - E\delta_{mm'}) C_{mm'} \quad 0 \quad . \tag{12}$$

The matrix elements in Eq. (12) are given by

$$H_{mm} = [N\hbar\omega_{-} + (N+n)\hbar\Omega - (N-m)\hbar\Delta] \tag{13}$$

$$H_{mm} = \hbar\lambda\sqrt{(n+m+1)(N-m)(m+1)}\ \delta_{m+1,m'}$$

$$+ \hbar\lambda\sqrt{(n+m'+1)(N-m')(m'+1)}\ \delta_{m,m'+1} \quad . \tag{14}$$

where the detuning parameter is defined by

$$\Delta = \Omega - (\omega_{+} - \omega_{-}) \quad . \tag{15}$$

The solution of Eq. (12) provides us with energy eigenvectors and corresponding eigenvalues

$$H|\phi_{n\sigma}\rangle = E_{n\sigma}|\phi_{n\sigma}\rangle \tag{16}$$

$$|\phi_{n\sigma}\rangle = \sum_{m=0}^{N} C^{\sigma}_{nm} |N-m, m\rangle |n+m\rangle \quad . \tag{17}$$

where σ labels the eigenvectors in this particular subspace. The orthonormality relation is given by

$$\sum_{m=0}^{N} C^{\sigma'}_{nm} C^{\sigma}_{nm} = \delta_{\sigma\sigma'} \quad . \tag{18}$$

It is important to note that if initially the atomic system is in a state $|N,0\rangle$ and the field in a linearly superimposed Fock state, then at any time $t > 0$ the atom-field system can only be found in one of the subspaces with the total excitaiton N, N+1, ..., N+n, ... This can be summarized by the completeness condition

$$\sum_{n=\sigma}^{\infty} \sum_{\sigma=1}^{N+1} |\phi_{n\sigma}\rangle\langle\phi_{n\sigma}| = 1 \quad . \tag{19}$$

The equation of motion for the matrix element of the density matrix can be obtained from making use of (17) and (19), namely,

$$\frac{\partial}{\partial t} \rho_{n\sigma,n'\sigma'}(t) = -\frac{i}{\hbar} (E_{n\sigma} - E_{n'\sigma'}) \rho_{n\sigma,n'\sigma'} \quad . \tag{20}$$

The solution of (20) can be put in the form

$$\rho_{n\sigma,n'\sigma'}(t) = \rho_{n\sigma,n'\sigma'}(0)\, e^{-\frac{i}{\hbar}(E_{n\sigma}-E_{n'\sigma'})t} \quad . \tag{21}$$

where the initial density matrix elements is given by

$$\rho_{n\sigma,n'\sigma'}(0) = \rho_{nn'}C^{-1}_{n\sigma}C^{-1}_{n'\sigma'} \quad . \tag{22}$$

Here, C^{-1} is the inverse matrix of the transformation defined in (11), and $\rho_{nn'}$ is the initial density matrix element of the field in the photon number representation.

Now the expectation value of any function $G(a^{\dagger},a)$ can be calculated by means of the density matrix in the usual manner, namely

$$\langle G\rangle = \mathrm{Tr}\{\rho G\} = \sum_{n}\sum_{\sigma} \rho_{n\sigma,n'\sigma'}(t)\,\langle \phi_{n'\sigma'}|G(a^{\dagger},a)|\phi_{n\sigma}\rangle \quad . \tag{23}$$

To study the squeezing of the field, we introduce as usual the slowly-varying complex amplitudes of the cavity field,

$$d_1 = \frac{1}{2}\{ae^{i\Omega t} + a^{\dagger}e^{-i\Omega t}\} \quad , \tag{24a}$$

$$d_2 = \frac{1}{2i}\{ae^{i\Omega t} - a^{\dagger}e^{-i\Omega t}\} \quad . \tag{24b}$$

These operators satisfy the commutation relation

$$[d_1,d_2] = \frac{i}{2} \quad , \tag{25}$$

which implies the uncertainty relation

$$\langle(\Delta d_1)^2\rangle\,\langle(\Delta d_2)^2\rangle \geq \frac{1}{16} \quad . \tag{26}$$

The electromagnetic field is squeezed to second order if $\langle(\Delta d_1)^2\rangle < \frac{1}{4}$ or $\langle(\Delta d_2)^2\rangle < \frac{1}{4}$.

As we have mentioned above, the concept of squeezing has been generalized to K-th order where K is limited to an even number[2] that is, the field is squeezed to K-th order if

$$\langle(\Delta d_{1,2})^K\rangle < (K-1)!!/2^K \quad . \tag{27}$$

In particular, the fourth-order variance is given by

$$\langle(\Delta d_i)^4\rangle = \langle d_i^4\rangle - 4\langle d_i\rangle\langle d_i^3\rangle + 6\langle d_i^2\rangle\langle d_i\rangle^2 - 3\langle d_i\rangle^4 \quad , \tag{28}$$

and fourth-order squeezing occurs whenever $\langle(d_1)^4\rangle < \frac{3}{16}$ or $\langle(\Delta d_2)^4\rangle < \frac{3}{16}$.

III. Second-order squeezing

Throughout this paper, we take $1/\lambda$ as the unit of time and set $\hbar = 1$ in our numerical work. For the initial condition, we have assumed in the above derivation that all atoms are in their upper states. We now assume further that the field is in the coherent state at $t = 0$. Thus

$$\rho_{nn'} = \frac{z^{n'} z^{*n}}{\sqrt{n!n'!}} e^{-\bar{n}} \quad , \tag{29}$$

where $\bar{n} = |z|^2$ is the initial mean photon number. For simplicity, we choose z to be real in this paper.

The mean square deviation of d_1 is computed as a function of time for different values of $\bar{n}$ and Δ, the detuning parameter. For the case of

resonance interaction, we have actually calculated $<(\Delta d_1)>^2$ for $\bar{n}$ = 5, 15, 25 and 35, and the results for $\bar{n}$ = 5 and 35 are plotted in Fig. 1. We observe that the squeezing becomes shallow as the number of atoms increases for $\bar{n}$ = 5 or for weak excitations. When N = 20, the squeezing disappears completely, as is clearly seen in Fig. 1(a). The situation is very different for stronger excitations. For $\bar{n} \geq 15$, we find that the maximum squeezing increases with increasing N after the field changes for the first time from the coherent state to the squeezing state. This is illustrated, as an example, in Fig. 1(b) for $\bar{n}$ = 35. Therefore, the influence of the atomic cooperative effect on second-order squeezing after the interaction is switched on is quite closely related to the initial intensity of the cavity field.

In Fig. 2, we plot the time evolution of $<(\Delta d_1)^2>$ for various $\bar{n}$ and N = 10. It is seen that during the first two squeezing periods, smaller $\bar{n}$ corresponds to deeper squeezing of the field. However, the opposite is true during the third squeezing period, and the squeezing deepens with increasing $\bar{n}$, although the strongest squeezing occurs in the second period. Thus the squeezing can not be increased arbitrarily by increasing the initial field intensity, even though it is possible to cause the field to return to the squeezing state more times.

The long-time behavior of $<(\Delta d_1)^2>$ is shown in Fig. 3 for one-atom and two-atom systems. We find that in the one-atom case the squeezing effect can be very strong at a much later time. As a matter of fact, it has been shown[11] that the squeezing increases with increasing $\bar{n}$. In the two-atom case, however, the field enters the squeezed state only briefly at a time shortly after the interaction with the atom takes place. Thus the cooperation among atoms changes the field squeezing property indeed. It is worth pointing out that although the second-order squeezing has been calculated in Ref. 8 with

the same model, no discussion of the long-time behavior or the effect due to the number of atoms was given in that work.

We have also investigated the relation between the cooperative effect and the detuning parameter. The minimum $\langle(\Delta d_1)^2\rangle$ is plotted versus Δ in Fig. 4 with fixed $\bar{n}$ for different numbers of atoms in the cavity. It shows that for a single-atom, there exists a non-zero Δ which leads to the strongest squeezing. In cases of more than one atom in the cavity, maximum squeezing occurs only at $\Delta = 0$. This means that the cooperative interaction between the atoms weakens the contribution of the virtual atomic transition to the squeezing of the cavity field.

IV. Fourth-order squeezing

To study fourth-order squeezing, we have calculated $\langle(\Delta d_1)^4\rangle$ as a function of time for $\bar{n} = 5$, 15, 25 and 35, and only the results for $\bar{n} = 5$ and 35 are presented here. Since there is no sign of squeezing in the long-time regime, we plot only the short-time behavior of the evolution.

Figure 5(a) shows no fourth-order squeezing at any time. Furthermore, the first minimum value of the curve increases with increasing N. This is very different from the one-atom case, in which the field can be found in the fourth-order squeezing state even if $\bar{n}$ is as small as 0.2.[3] This indicates that the atomic cooperative effect due to the interaction with the cavity field tends to increase the fourth-order fluctuation of the field when the initial field intensity is weak. When the initial field intensity is strong, as shown in Fig. 5(b) for $\bar{n} = 35$, the first squeezing deepens with increasing N. Thus the atomic cooperative effect will reduce the fourth-order fluctuation of the field after the interaction is switched on. Therefore, the atomic cooperative effect on the field squeezing depends strongly upon the

initial field strength in the cavity. We also note that for a given $\bar{n}$, the field enters the squeezed state from the initial coherent state at about the same time, and this time is insensitive to the number of atoms in the cavity.

In order to study the dependence of the squeezing on the initial field excitation, we plot in Fig. 6 the time evolution of $\langle(\Delta d_1)^4\rangle$ for a given N but different $\bar{n}$. It is clearly seen from these curves that the first squeezing becomes shallower as $\bar{n}$ increases, while the third squeezing deepens with increasing $\bar{n}$. In the case of many atoms in a cavity, it is not possible to increase the squeezing arbitrarily by increasing the initial field intensity. By so doing one can only increase the number of times the field becomes squeezed.

The influence of the variation in the detuning parameter on the fourth-order squeezing can be changed by the cooperation of atoms in the cavity. In Fig. 7, we plot the minimum of $\langle(\Delta d_1)^4\rangle$ versus the detuning for various N. Like the second-order squeezing, we find that the atomic cooperation changes the dependence of the squeezing on the detuning; that is, the squeezing reduces with increasing detuning when there is more than one atom in the cavity. On the other hand, as curve a indicates, the squeezing can be greatly increased by suitably choosing the detuning in the case of a single atom in the cavity. It is also noted that similar to the situation in Fig. 3(a) for the one-atom case, strong fourth-order squeezing occurs in the long-time regime. This pheneomena disappears totally, however, for $N \geq 2$. Finally, it should be of interest to point out that fourth-order squeezing is not intrinsic to the system in the present model because it always accompanies second-order squeezing in this model.

V. Discussion

We have studied higher-order squeezing of the cavity field which is interacting with N two-level atoms by means of the SU(2) Lie algebra. The method of approach is similar to what we have applied in our analysis of an M-level atom interacting with cavity fields.[12] Detailed calculations reveal that the initial field intensity plays a key role in the determination of the influence of the many-body cooperation on the field squeezing. For strong initial excitations, the atomic cooperation enhances the squeezing, while the opposite is true when the initial excitation is weak.

In a micromaser,[6] inversely populated atoms are injected one by one into an optical cavity to create stimulated emission. We suggest that a many-atom maser may be designed in which a group of active atoms instead of one is injected into the cavity at one time. If such many-atom micromasers are realized, it should be much easier to observe the field squeezing with strong initial excitation. Furthermore, it is known that a micromaser may be expected to produce low-noise microwave radiation. If the active medium consists of only one atom, its output power is too small to be useful for any practical purpose. If, however, the active medium consists of a group of atoms, or a large number of atoms is allowed to pass the cavity at one time, the output power of the micromaser will be greatly increased. The investigation of a many-atom micromaser is being carried out and will be reported in the future.

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Appendix: Derivation of Eq. (8)

To derive the state vector for an arbitrary number of atoms in the upper level, we start with the initial state

$$|N,0\rangle = \prod_{i=1}^{N} A_+^\dagger(i)|0\rangle \quad , \tag{A1}$$

where $|0\rangle$ is the vacuum state defined by

$$\begin{aligned} A_+(i)|0\rangle &= 0 \\ A_-(i)|0\rangle &= 0 \end{aligned} \qquad i = 1,2,\ldots,N \quad . \tag{A2}$$

$|N,0\rangle$ is the highest-weight state from which we can obtain any arbitrary state specified by the total number of atoms N in the SU(2) space of the full symmetry representation by applying the lowering operator D repeatedly. Thus we have, after m applications of the operator D, $DD\ldots D|N,0\rangle = D^m|N,0\rangle$.

From the commutators

$$\begin{aligned} [N_+,D] &= -D \\ [N_-,D] &= D \end{aligned} \tag{A3}$$

we can easily find the recursion formula

$$\begin{aligned} N_+D^m &= D^mN_+ - mD^m \\ N_-D^m &= D^mN_- + mD^m \quad . \end{aligned} \tag{A4}$$

With the help of the definitions of number operators

$$N_+|N,0\rangle = N|N,0\rangle \tag{A5}$$

$$N_-|N,0\rangle = 0 \quad ,$$

it follows from Eqs. (A4) that

$$\begin{aligned} N_+ D^m|N,0\rangle &= (N-m)D^m|N,0\rangle \\ N_- D^m|N,0\rangle &= mD^m|N,0\rangle \quad . \end{aligned} \tag{A6}$$

This means that there are N-m atoms in the upper level and m atoms in the lower level in the state $D^m|N,0\rangle$; that is,

$$D^m|N,0\rangle \propto |N-m,m\rangle \quad , \qquad 0 \leq m \leq N \quad . \tag{A7}$$

Equation (A7) represents the set of N+1 basis vectors that span the SU(2) space of full symmetry representation corresponding to states of N atoms.

The normalization constant C can be determined in the following manner. Since

$$D^\dagger|N,0\rangle = A_+^\dagger A_-|N,0\rangle = 0 \quad , \tag{A8}$$

we have

$$\begin{aligned} C^2 &= \langle 0,N|(D^\dagger)^m D^m|N,0\rangle \\ &= \begin{cases} 1, & m = 0 \\ (N-m+1)m\langle 0,N|(D^\dagger)^{m-1}D^{m-1}|N,0\rangle, & m \geq 1 \end{cases} \end{aligned} \tag{A9}$$

where we have made use of the commutation relation

$$[D,D^{\dagger}] = N_{-} - N_{+} \quad . \tag{A10}$$

Equation (A9) implies that

$$C^2 = N!\cdot m!/(N-m)! \quad . \tag{A11}$$

Combining (A11) with (A7), we obtain

$$|N-m,m\rangle = \sqrt{\frac{(N-m)!}{N!m!}}\, D^m |N,0\rangle \quad , \tag{A12}$$

which is just Eq. (8).

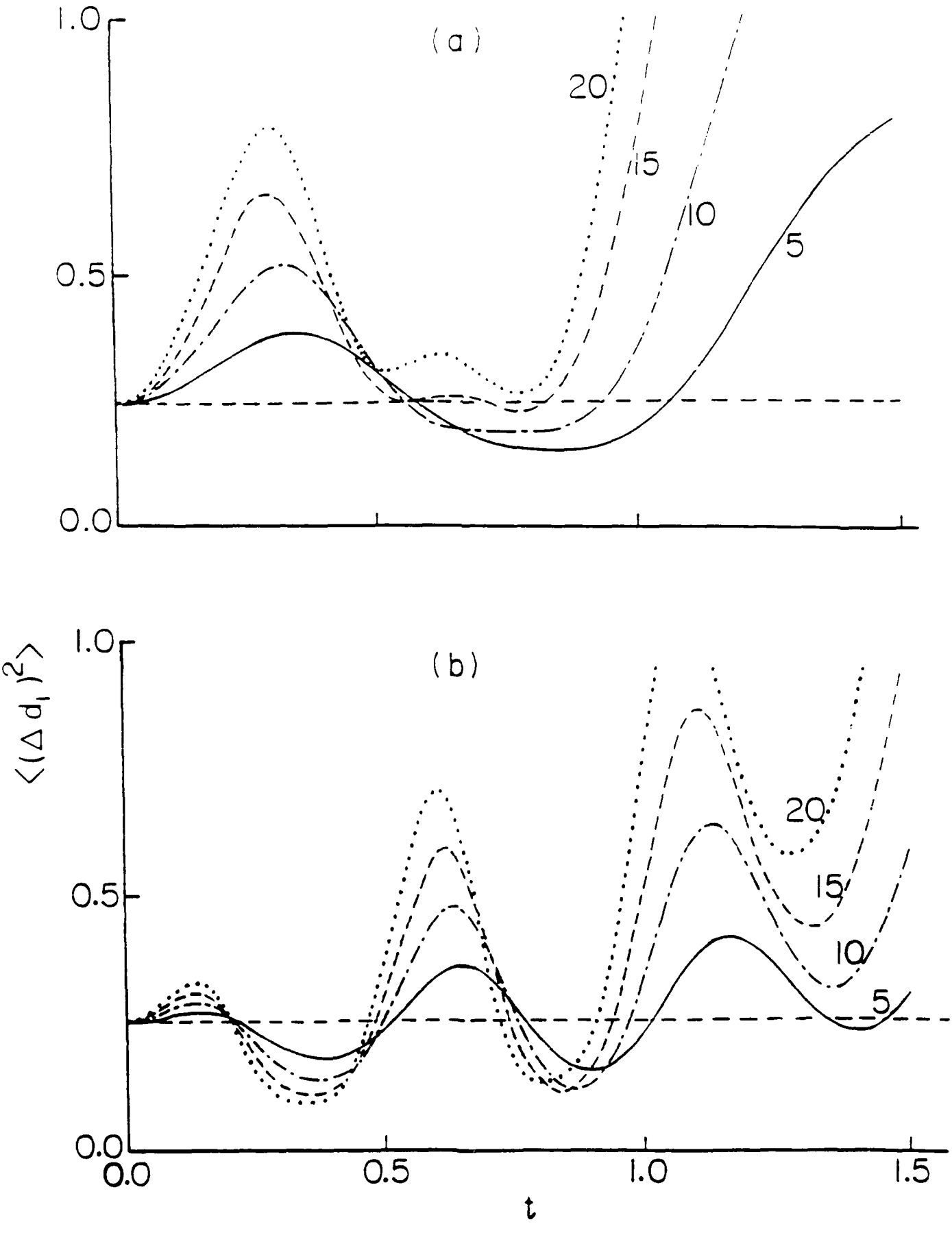
References

1. H. P. Yuen, Phys. Rev. A 13, 2226 (1976).
2. C. K. Hong and L. Mandel, Phys. Rev. Lett. 54, 323 (1985); C. K. Hong and L. Mandel, Phys. Rev. A 32, 974 (1985).
3. M. Kozierowski, Phys. Rev. A 34, 3474 (1986).
4. F. L. Kien, M. Kozierowski and T. Quang, Phys. Rev. A 38, 263 (1988); L. Christopher, C. Gerry and P. J. Moyer, Phys. Rev. A 38, 5665 (1988).
5. C. Gerry and S. Rodriques, Phys. Rev. A 35, 4440 (1987).
6. D. Meschede, H. Walther and G. Muller, Phys. Rev. Lett. 54, 551 (1985); G. Rempe, H. Walther and N. Klein, Phys. Rev. Lett. 58, 353 (1987).
7. S. M. Barnett and P. L. Knight, Optica Acta 31, 435 (1984).
8. R. H. Dicke, Phys. Rev. 93, 99 (1954).
9. J. Seke and F. Rattay, J. Opt. Soc. Am. B 4, 380 (1987); Phys. Rev. A 39, 171 (1989).
10. M. Butler and D. D. Drummond, Optica Acta 33, 1 (1986).
11. J. R. Kuklinsky and J. L. Madajczyk, Phys. Rev. A 37, 3175 (1988).
12. F. L. Li, D. L. Lin, T. F. George and X. S. Li, Phys. Rev. A 40, 1394 (1989).

Figure Captions

1. Time evolution of $<(\Delta d_1)^2>$ for different numbers N of atoms (as indicated by the number next to each curve) and the detuning $\Delta = 0$. (a) $\bar{n} = 5$ (b) $\bar{n} = 35$.
2. Time evolution of $<(\Delta d_1)^2>$ for various choices of the initial mean photon number $\bar{n}$ (as indicated by the number next to each curve). Other parameters are $N = 10$ and $\Delta = 0$.
3. Long-time behavior of $<(\Delta d_1)^2>$ for $\bar{n} = 35$ and $\Delta = 0$. (a) $N = 1$ (b) $N = 2$.
4. Minimum $<(\Delta d_1)^2>$ versus the detuning for $\bar{n} = 25$. (a) $N = 1$, (b) $N = 2$, (c) $N = 5$.
5. Time evolution of $<(\Delta d_1)^4>$ for different numbers N of atoms (as incidated by the number next to each curve) and $\Delta = 0$. (a) $\bar{n} = 5$, (b) $\bar{n} = 35$.
6. Time evolution of $<(\Delta d_1)^4>$ for various choices of the initial mean photon number $\bar{n}$ (as indicated by the number next to each curve). Other parameters are $N = 10$ and $\Delta = 0$.
7. Minimum $<(\Delta d_1)^4>$ versus detuning for $\bar{n} = 25$. (a) $N = 1$, (b) $N = 2$, (c) $N = 5$.

Fig. 1


(a)
20
15
10
5
1.0
0.5
0.0
(b)
$\langle(\Delta d_1)^2\rangle$
20
15
10
5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
t

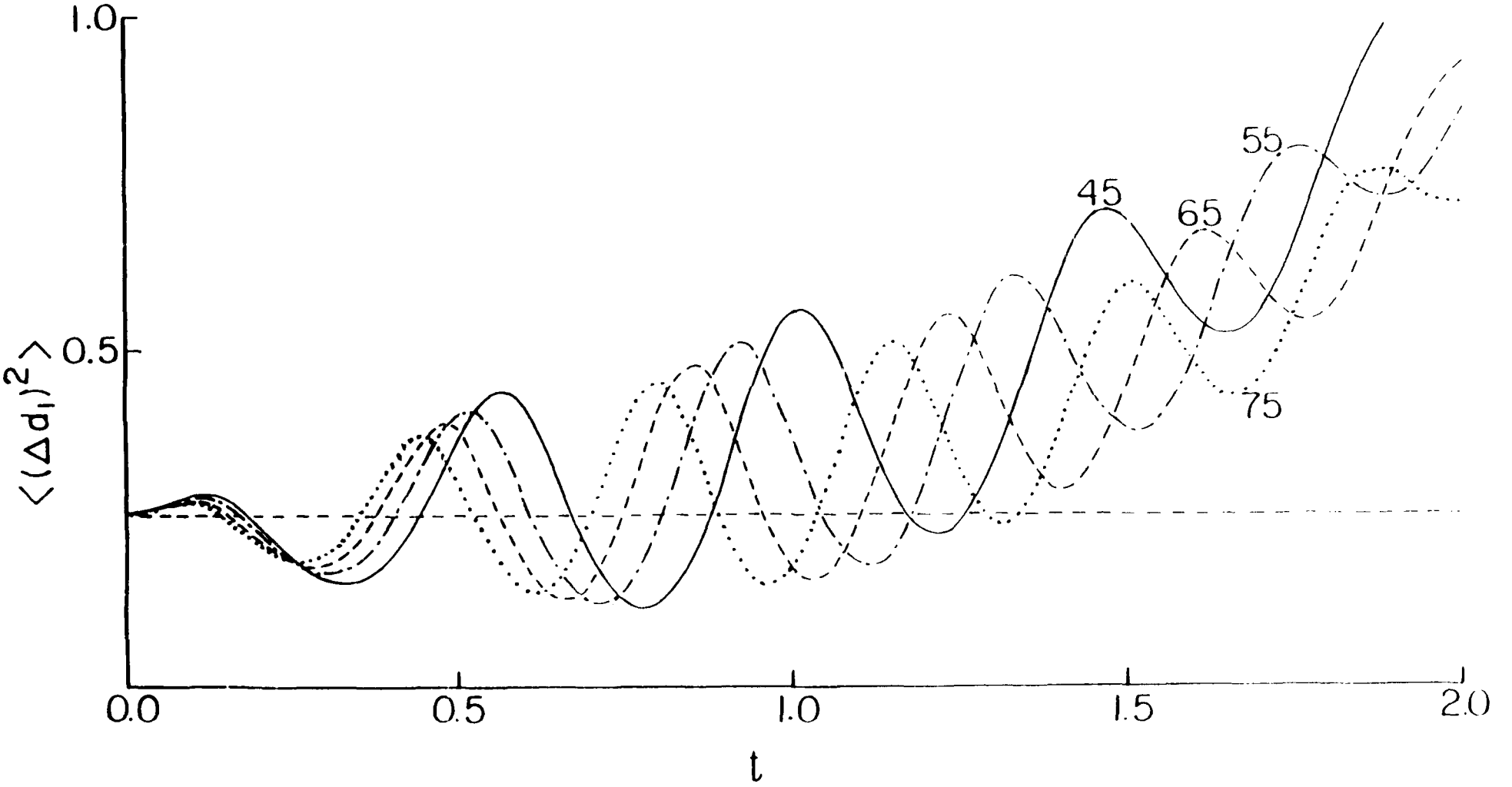
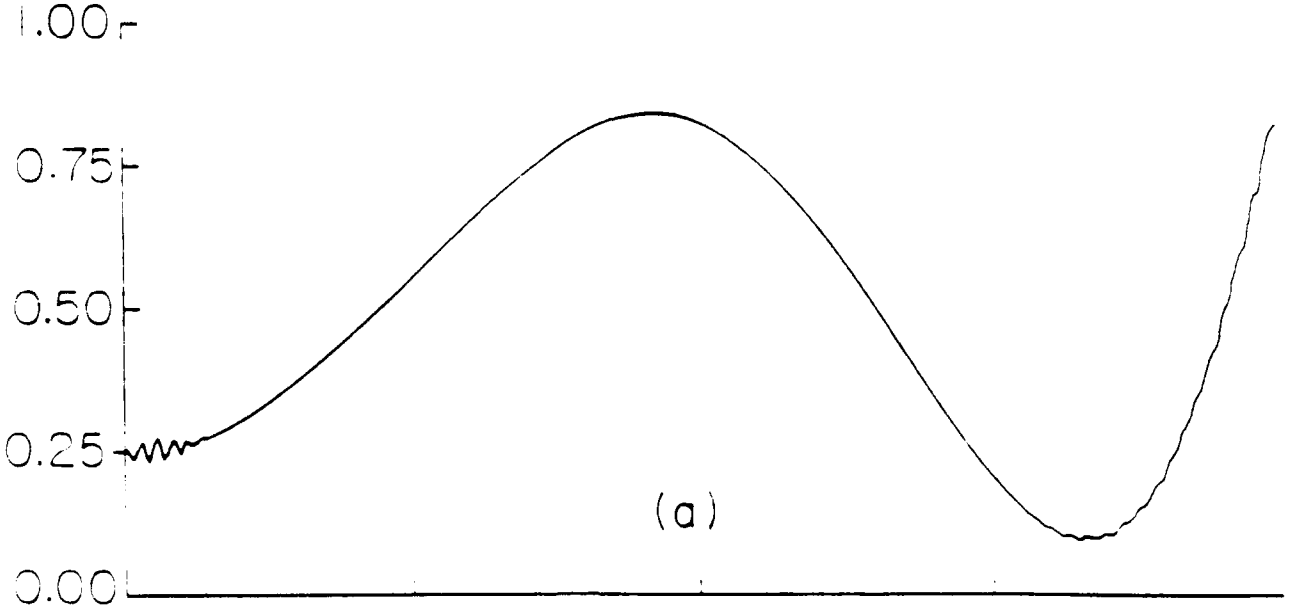
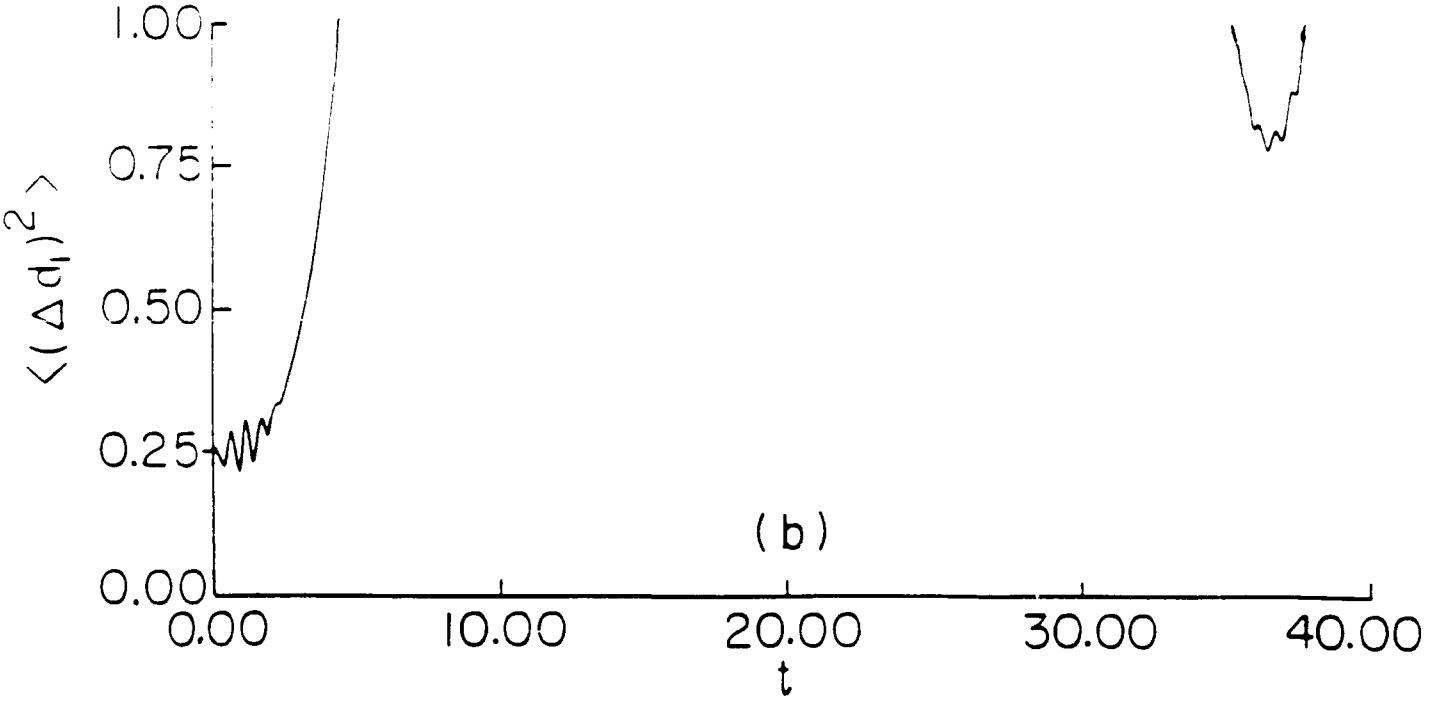


Fig. 1

Fig. 3


1.00
0.75
0.50
0.25
0.00
(a)


1.00
0.75
0.50
0.25
0.00
$\langle(\Delta d_1)^2\rangle$
(b)
0.00
10.00
20.00
30.00
40.00
t

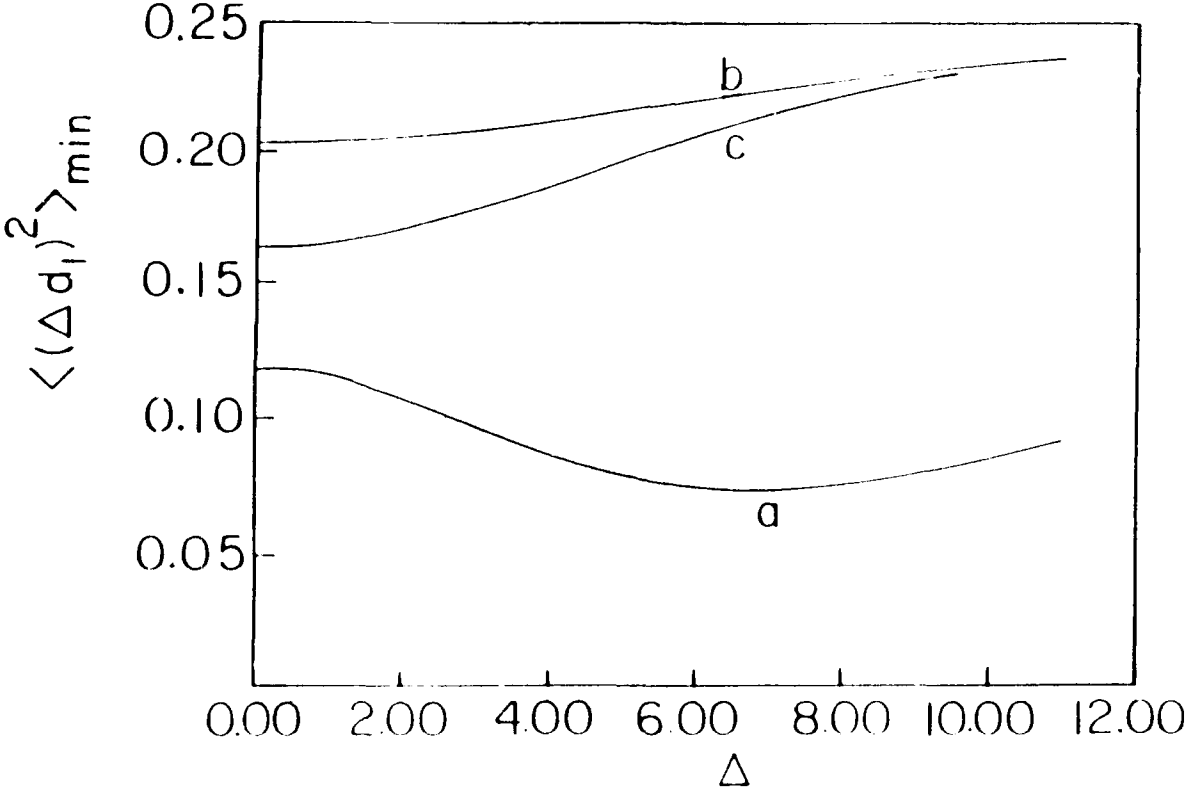
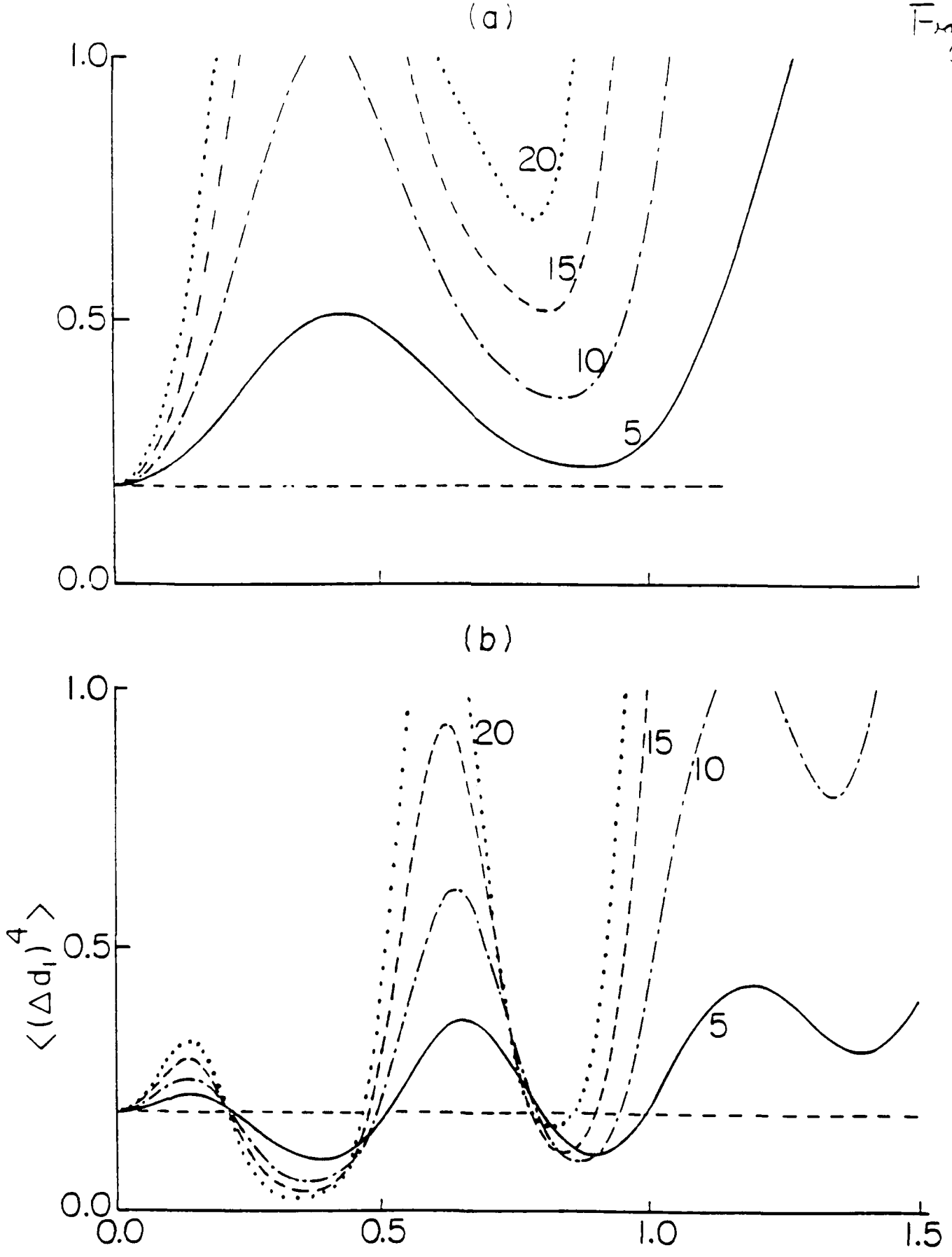
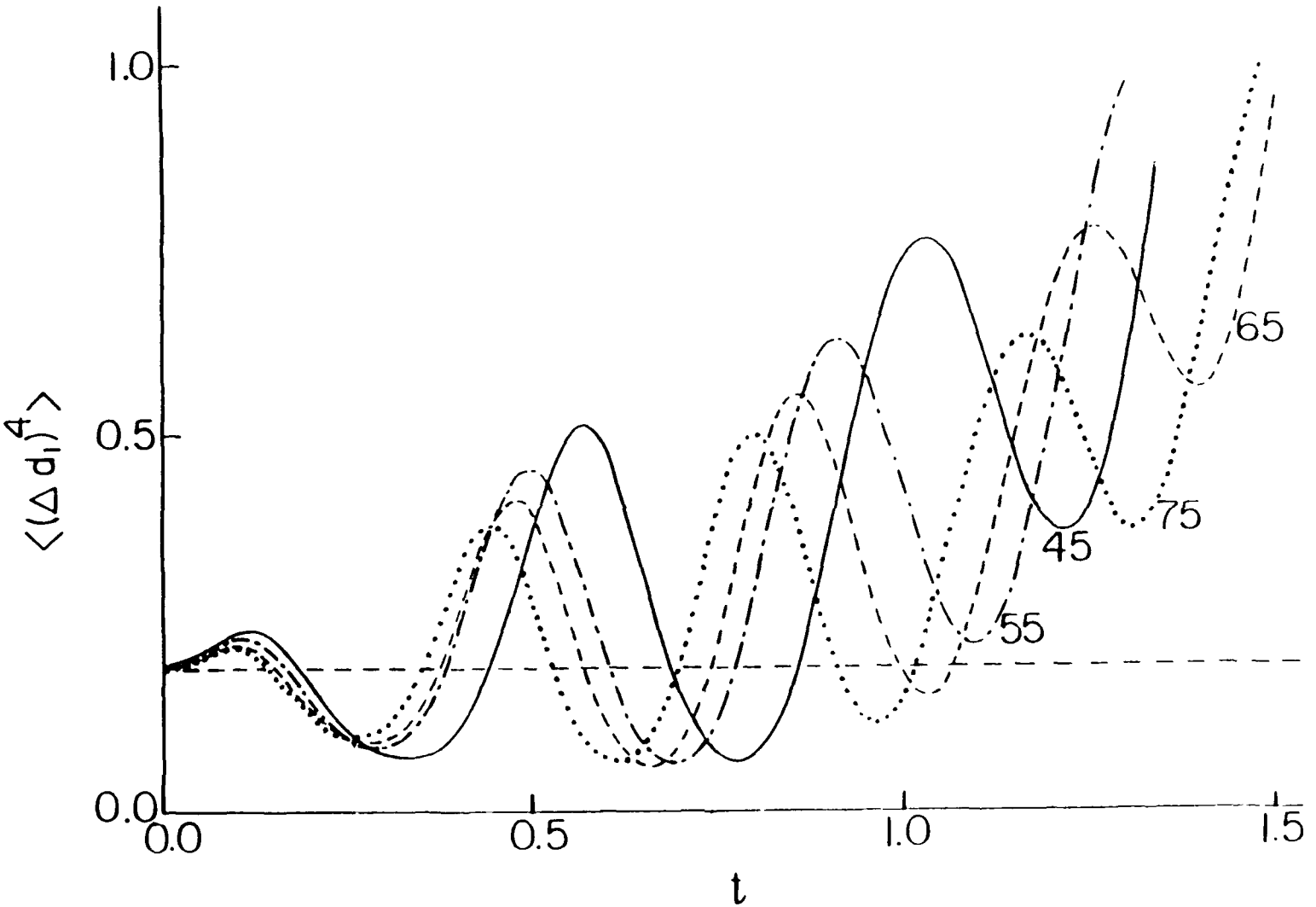
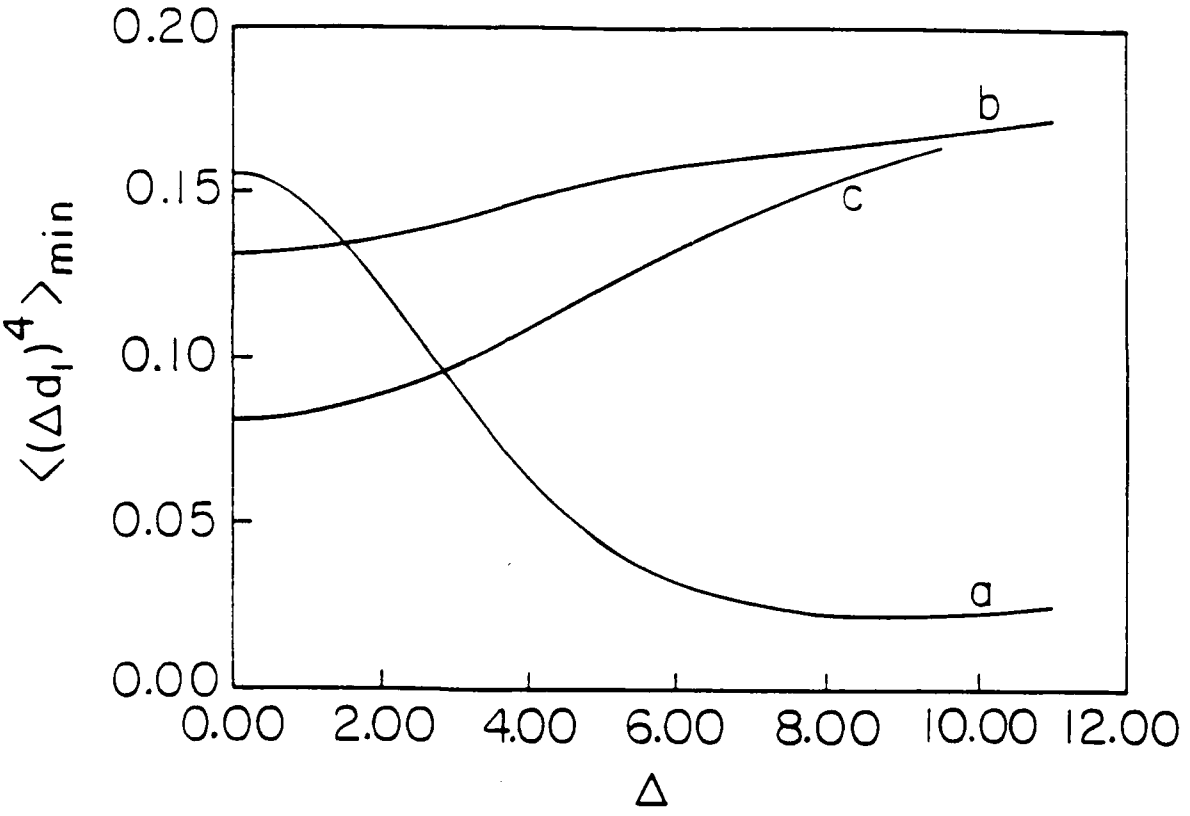

0.25
0.20
0.15
0.10
0.05
$\langle(\Delta d_i)^2\rangle_{min}$
b
c
a
0.00
2.00
4.00
6.00
8.00
10.00
12.00
Δ

Fig. 5


(a)
1.0
0.5
0.0
20
15
10
5
(b)
1.0
0.5
0.0
$\langle (\Delta d_1)^4 \rangle$
20
15
10
5
0.0
0.5
1.0
1.5


1.0
0.5
0.0
0.0
0.5
1.0
1.5
t
$\langle(\Delta d_1)^4\rangle$
45
55
65
75


0.20
0.15
0.10
0.05
0.00
$\langle(\Delta d_1)^4\rangle_{min}$
b
c
a
0.00
2.00
4.00
6.00
8.00
10.00
12.00
Δ

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